# Quantities and Relationships

# 1

**1.1** **A Picture Is Worth a Thousand Words**
Understanding Quantities and Their Relationships . . . . . 3

**1.2** **A Sort of Sorts**
Analyzing and Sorting Graphs . . . . . 17

**1.3** **There Are Many Ways to Represent Functions**
Recognizing Algebraic and Graphical Representations of Functions . . . . . 35

**1.4** **Function Families for 200, Alex...**
Recognizing Functions by Characteristics . . . . . 53


© 2012 Carnegie Learning

© 2012 Carnegie Learning

1.1

# A Picture Is Worth a Thousand Words

## Understanding Quantities and Their Relationships

### LEARNING GOALS

In this lesson, you will:

- Understand quantities and their relationships with each other.
- Identify the independent and dependent quantities for a problem situation.
- Match a graph with an appropriate problem situation.
- Label the independent and dependent quantities on a graph.
- Review and analyze graphs.
- Describe similarities and differences among graphs.

### KEY TERMS

- dependent quantity
- independent quantity

How interesting would a website be without pictures or illustrations? Does an inviting image on a magazine cover make you more likely to buy it? Pictures and images aren't just for drawing your attention, though. They also bring life to text and stories.

There is an old proverb that states that a picture is worth a thousand words. There is a lot of truth in this saying—and images have been used by humans for a long time to communicate. Just think: would you rather post a story of your adventure on a social media site, or post one picture to tell your thousand-word story in a glance?

© 2012 Carnegie Learning

1

## PROBLEM 1 What's the Dependency?

Have you ever planned for a party? You may have purchased ice, gone grocery shopping, selected music, made food, or even cleaned in preparation. Many times, these tasks depend on another task being done first. For instance, you wouldn't make food before grocery shopping, now would you?

Let's consider the relationship between:

- the number of hours worked and the money earned.
- your grade on a test and the number of hours you studied.
- the number of people working on a particular job and the time it takes to complete a job.
- the number of games played and the number of points scored.
- the speed of a car and how far the driver pushes down on the gas pedal.

There are two quantities that are changing in each situation. When one quantity depends on another in a problem situation, it is said to be the **dependent quantity**. The quantity that the dependent quantity depends upon is called the **independent quantity**.

1. Circle the independent quantity and underline the dependent quantity in each statement.

2. Describe how you can determine which quantity is the independent quantity and which quantity is the dependent quantity in any problem situation.

© 2012 Carnegie Learning

3. Read each scenario and then determine the independent and dependent quantities. Be sure to include the appropriate units of measure for each quantity.

### Something's Fishy

Candice is a building manager for the Crowley Enterprise office building. One of her responsibilities is cleaning the office building's 200-gallon aquarium. For cleaning, she must remove the fish from the aquarium and drain the water. The water drains at a constant rate of 10 gallons per minute.

- independent quantity:

- dependent quantity:

### Smart Phone, but Is It a Smart Deal?

You have had your eye on an upgraded smart phone. However, you currently do not have the money to purchase it. Your cousin will provide the funding, as long as you pay him interest. He tells you that you only need to pay $1 in interest initially, and then the interest will double each week after that. You consider his offer and wonder: is this *really* a good deal?

- independent quantity:

- dependent quantity:

© 2012 Carnegie Learning

1

### Can't Wait to Hit the Slopes!

Andrew loves skiing—he just hates the ski lift ride back up to the top of the hill. For some reason the ski lift has been acting up today. His last trip started fine. The ski lift traveled up the mountain at a steady rate of about 83 feet per minute. Then all of a sudden it stopped and Andrew sat there waiting for 10 minutes! Finally, the ski lift began to ascend up the mountain to the top.

- independent quantity:

- dependent quantity:

### It's Magic

The Amazing Aloysius is practicing one of his tricks. As part of this trick, he cuts a rope into many pieces and then magically puts the pieces of rope back together. He begins the trick with a 20-foot rope and then cuts it in half. He then takes one of the halves and cuts that piece in half. He repeats this process until he is left with a piece so small he can no longer cut it. He wants to know how many total cuts he can make and the length of each remaining piece of rope after the total number of cuts.

- independent quantity:

- dependent quantity:

© 2012 Carnegie Learning

### Baton Twirling

Jill is a drum major for the Altadena High School marching band. She has been practicing for the band's halftime performance. For the finale, Jill tosses her baton in the air so that it reaches a maximum height of 22 feet. This gives her 2 seconds to twirl around twice and catch the baton when it comes back down.

- independent quantity:

- dependent quantity:

### Music Club

Jermaine loves music. He can lip sync almost any song at a moment's notice. He joined *Songs When I Want Them*, an online music store. By becoming a member, Jermaine can purchase just about any song he wants. Jermaine pays $1 per song.

- independent quantity:

- dependent quantity:

© 2012 Carnegie Learning

1

### A Trip to School

On Monday morning, Myra began her 1.3-mile walk to school. After a few minutes of walking, she walked right into a spider's web—and Myra hates spiders! She began running until she ran into her friend Tanisha. She stopped and told Tanisha of her adventurous morning and the icky spider's web! Then they walked the rest of the way to school.

- independent quantity:

- dependent quantity:

### Jelly Bean Challenge

Mr. Wright judges the annual Jelly Bean Challenge at the summer fair. Every year, he encourages the citizens in his town to guess the number of jelly beans in a jar. He keeps a record of everyone's guesses and the number of jelly beans that each person's guess was off by.

- independent quantity:

- dependent quantity:

© 2012 Carnegie Learning

## PROBLEM 2 Matching Graphs and Scenarios

While a person can describe the monthly cost to operate a business, or talk about a marathon pace a runner ran to break a world record, graphs on a coordinate plane enable people to *see* the data. Graphs relay information about data in a visual way. If you read almost any newspaper, especially in the business section, you will probably encounter graphs.

Points on a coordinate plane that are or are not connected with a line or smooth curve model, or represent, a relationship in a problem situation. In some problem situations, all the points on the coordinate plane will make sense. In other problem situations, not all the points will make sense. So, when you model a relationship on a coordinate plane, it is up to you to consider the situation and interpret the meaning of the data values shown.

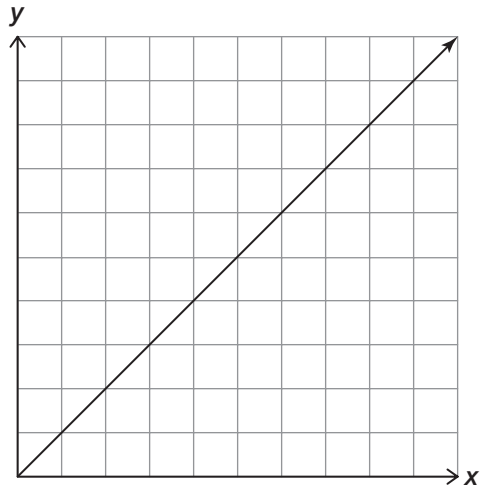
1. Cut out each graph on the following pages. Then, analyze each graph, match it to a scenario, and tape it next to the scenario it matches. For each graph, label the *x*- and *y*-axes with the appropriate quantity and unit of measure. Then, write the title of the problem situation on each graph.

© 2012 Carnegie Learning

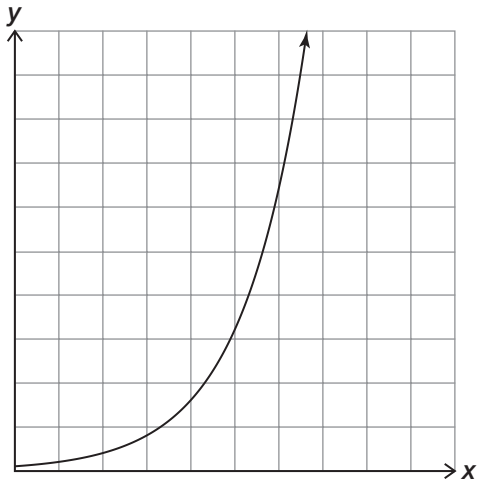
1

© 2012 Carnegie Learning

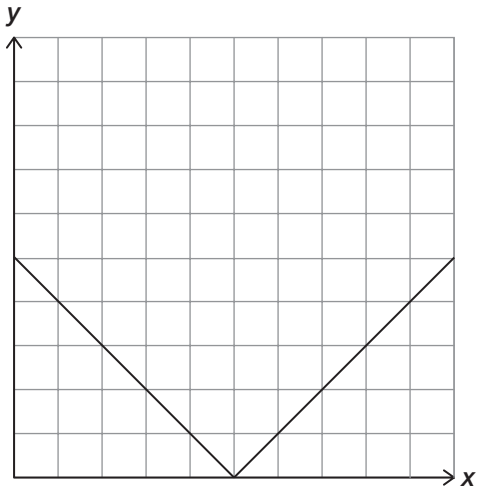
**Graph A**



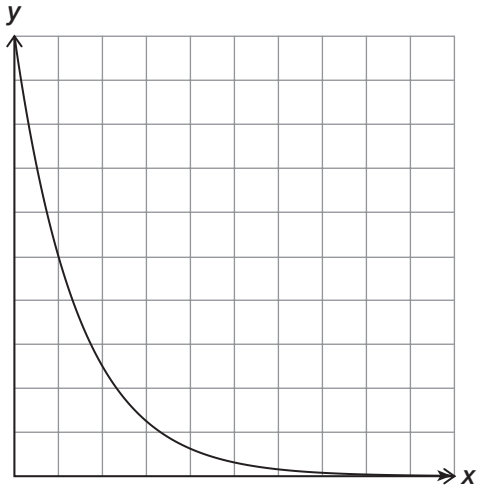

**Graph B**




**Graph C**




**Graph D**

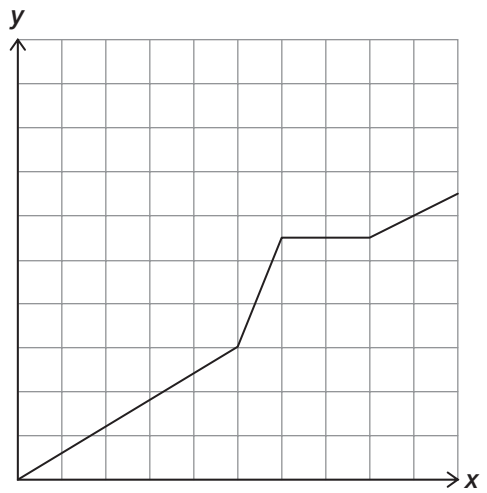



© 2012 Carnegie Learning

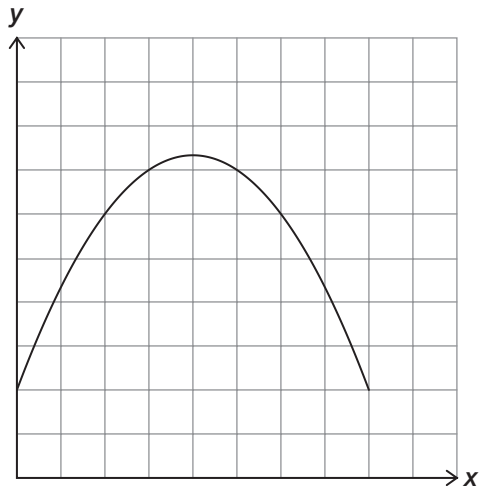
1

© 2012 Carnegie Learning

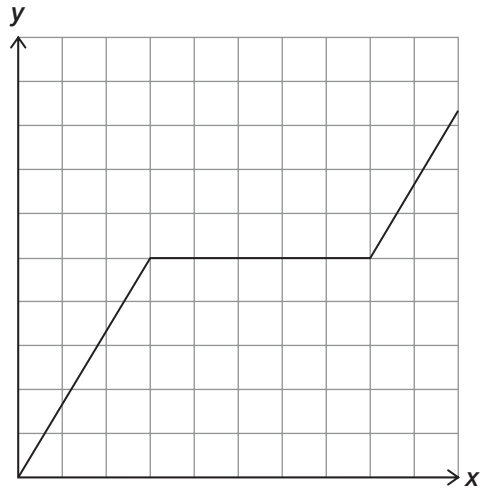
**Graph E**



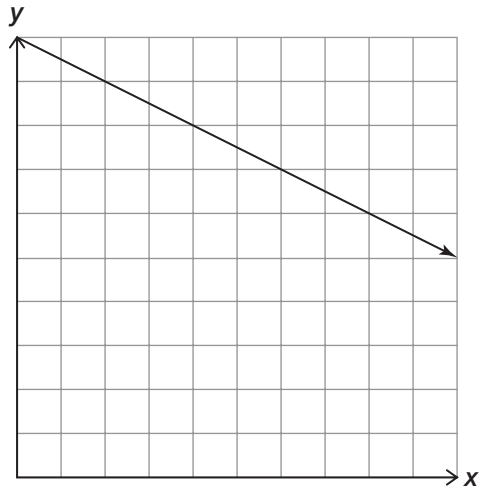

**Graph F**




**Graph G**




**Graph H**




© 2012 Carnegie Learning

1

© 2012 Carnegie Learning

## PROBLEM 3 Oh, Say, Can You See (in the Graphs)!

Now that you have matched a graph with the appropriate problem situation, let's go back and examine all the graphs.

1. What similarities do you notice in the graphs?

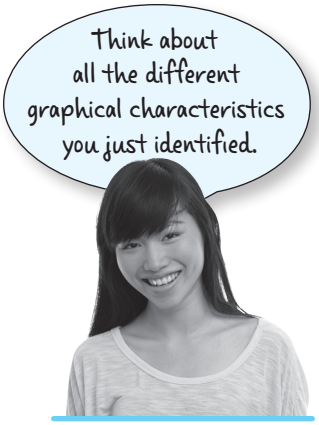
2. What differences do you notice in the graphs?

3. How did you label the independent and dependent quantities in each graph?

4. Analyze each graph from left to right. Describe any graphical characteristics you notice.

© 2012 Carnegie Learning

**5.** Compare the graphs for each scenario given and describe any similarities and differences you notice.




**a.** *Smart Phone, but Is It a Smart Deal?* and *Music Club*

**b.** *Something's Fishy* and *It's Magic*

**c.** *Baton Twirling* and *Jelly Bean Challenge*

**6.** Consider the scenario *A Trip to School*.

**a.** Write a scenario and sketch a graph to describe a possible trip on a different day.

**Scenario**

**Graph**

**b.** Compare your scenario and sketch with your classmates' scenarios and sketches. What similarities do you notice? What differences do you notice?

Be prepared to share your solutions and methods.

© 2012 Carnegie Learning

1.2

# A Sort of Sorts

## Analyzing and Sorting Graphs

### LEARNING GOALS

In this lesson, you will:

- Review and analyze graphs.
- Determine similarities and differences among various graphs.
- Sort graphs by their similarities and rationalize the differences between the groups of graphs.
- Use the Vertical Line Test to determine if the graph of a relation is a function.

### KEY TERMS

- relation
- domain
- range
- function
- Vertical Line Test
- discrete graph
- continuous graph

Are you getting the urge to start driving? Chances are that you'll be studying for your driving test before you know it. But how much will driving cost you? For all states in the U.S., auto insurance is a must before any driving can take place. For most teens and their families, this more than likely means an increase in auto insurance costs.

So how do insurance companies determine how much you will pay? The fact of the matter is that auto insurance companies sort drivers into different groups to determine their costs. For example, they sort drivers by gender, age, marital status, and driving experience. The type of car is also a factor. A sports vehicle or a luxury car is generally more expensive to insure than an economical car or a family sedan. Even the color of a car can affect the cost to insure it!

Do you think it is good business practice to group drivers to determine auto insurance costs? Or do you feel that each individual should be reviewed solely on the merit of the driver based on driving record? Do you think auto insurance companies factor in where a driver lives when computing insurance costs?

© 2012 Carnegie Learning

1

## PROBLEM 1 Let's Sort Some Graphs

Mathematics is the science of patterns and relationships. Looking for patterns and sorting objects into different groups can provide valuable insights. In this lesson, you will analyze many different graphs and sort them into various groups.

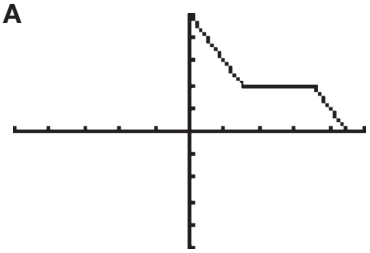
1. Cut out the twenty-two graphs on the following pages. Then analyze and sort the graphs into different groups. You may group the graphs in any way you feel is appropriate. However, you must sort the graphs into more than one group!

   In the space provided, record the following information for each of your groups.

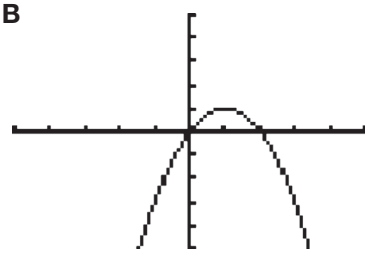
   - Name each group of graphs.
   - List the letters of the graphs in each group.
   - Provide a rationale why you created each group.

© 2012 Carnegie Learning

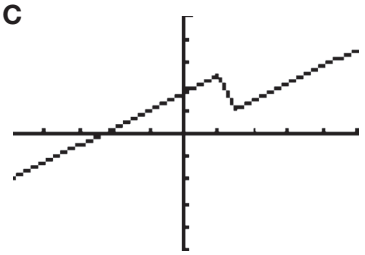
A



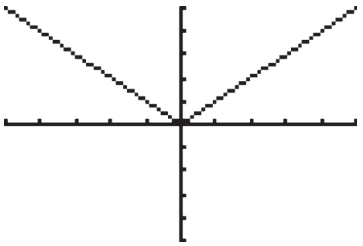
B



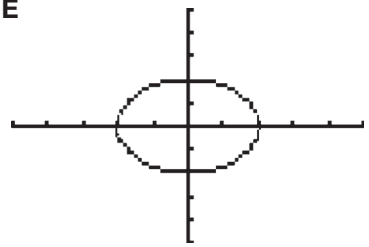
C



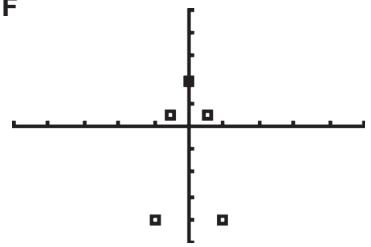
D



E



F

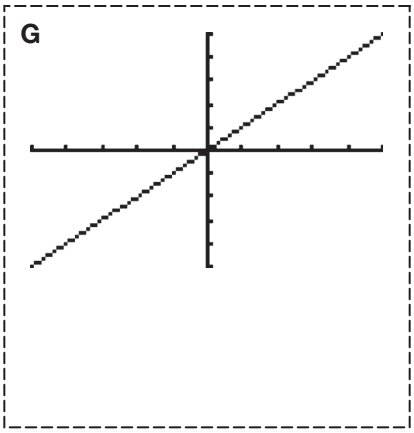


© 2012 Carnegie Learning

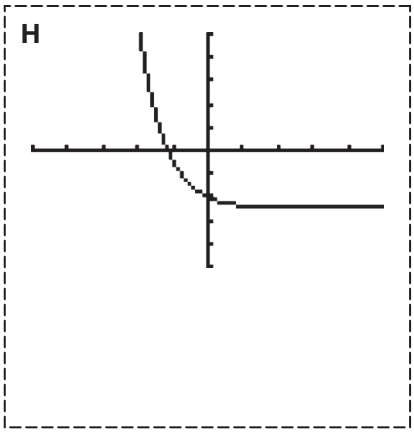
1

© 2012 Carnegie Learning

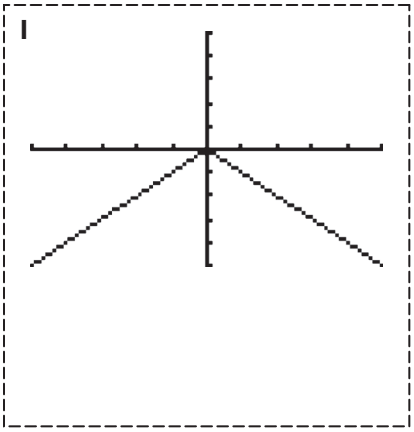
G



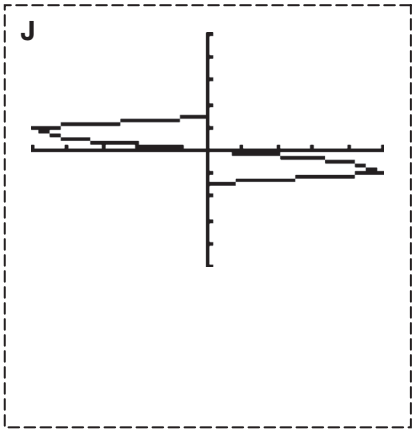
H



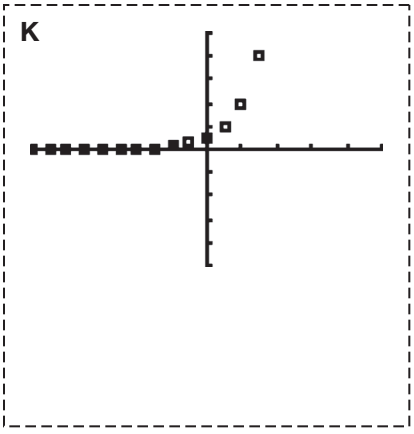
I



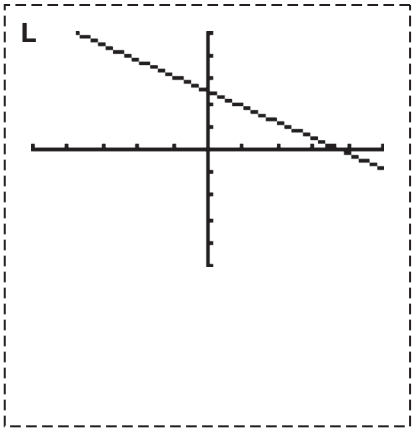
J



K



L

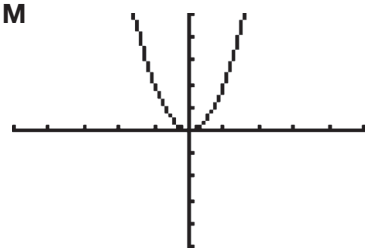


© 2012 Carnegie Learning

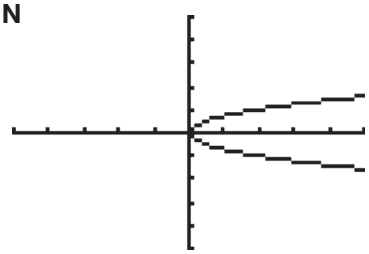
1

© 2012 Carnegie Learning

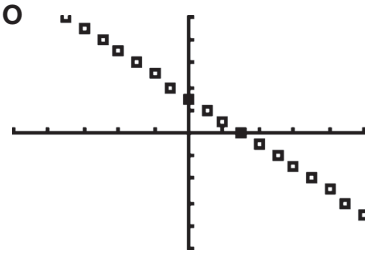
M



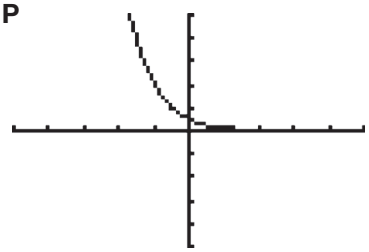
N



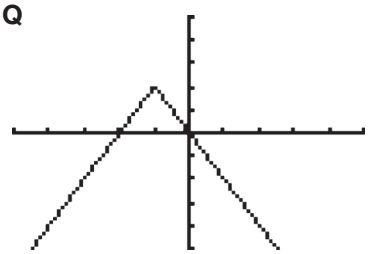
O



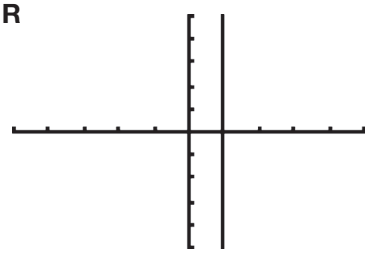
P



Q



R

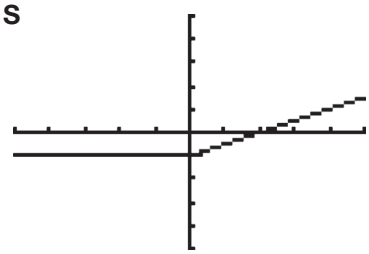


© 2012 Carnegie Learning

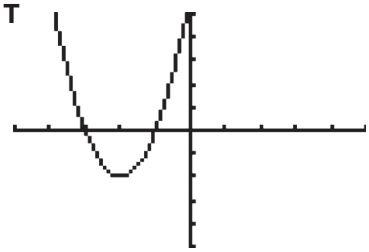
1

© 2012 Carnegie Learning

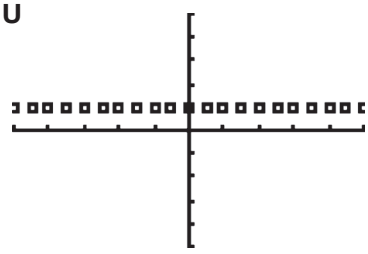
S



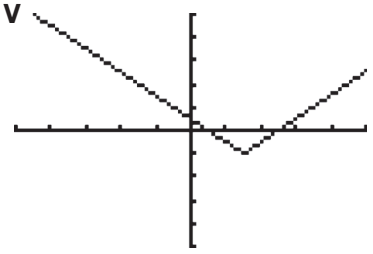
T



U



V



© 2012 Carnegie Learning

1

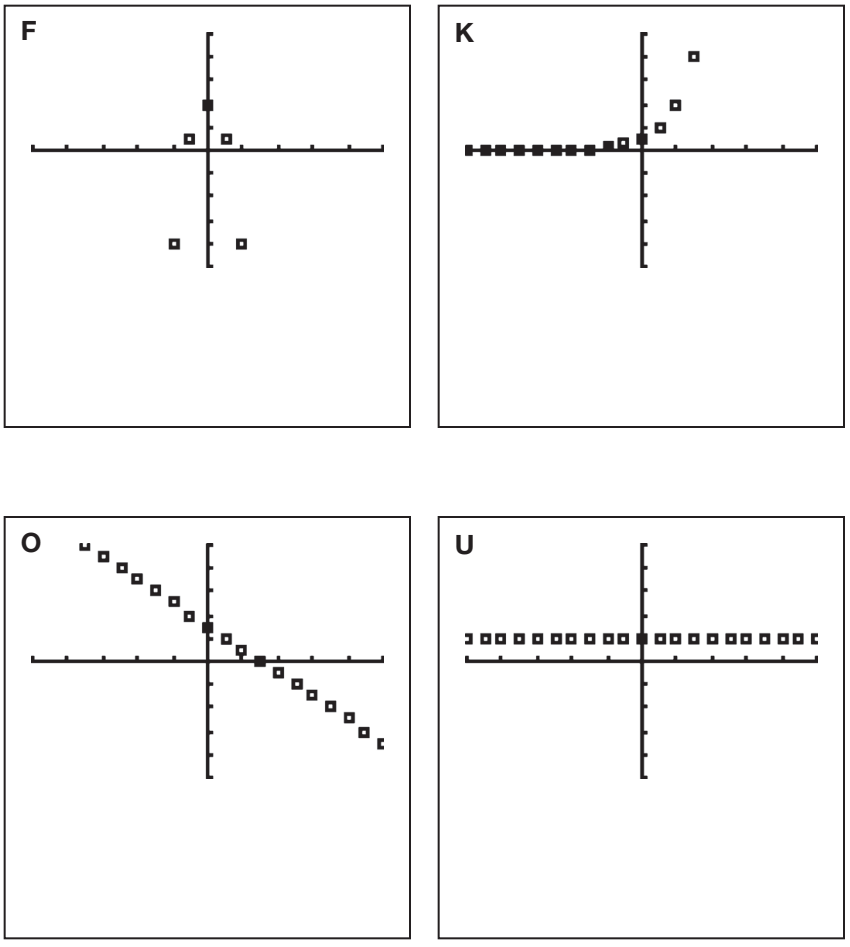
© 2012 Carnegie Learning

2. Compare your groupings with your classmates' groupings. Create a list of the different graphical behaviors you noticed.

© 2012 Carnegie Learning

## PROBLEM 2 I Like the Way You Think

**1.** Matthew grouped these graphs together.



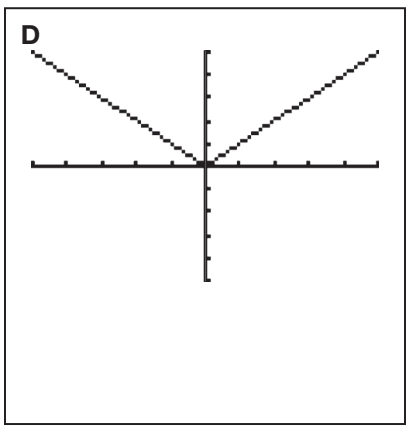

Why do you think Matthew put these graphs in the same group?

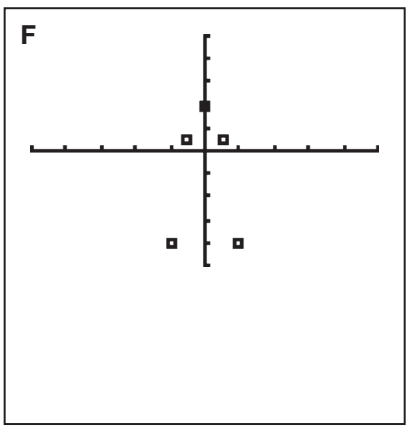
© 2012 Carnegie Learning

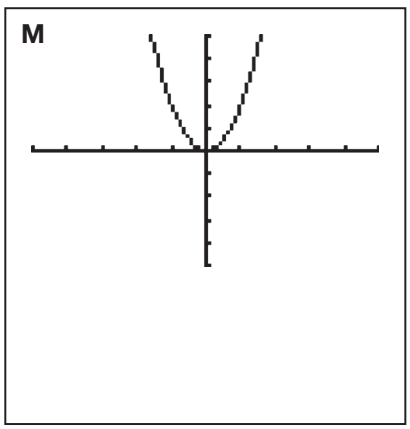
**2.**

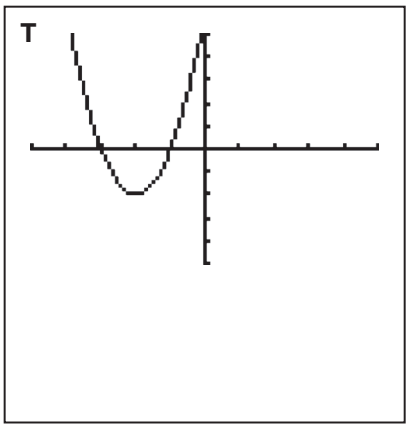
**Ashley**

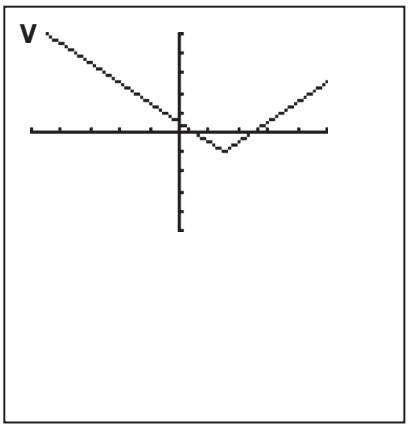
*I grouped these graphs together because they all show vertical symmetry. If I draw a vertical line through the middle of the graph, the image is the same on both sides.*




**a.** Show why Ashley's reasoning is correct.

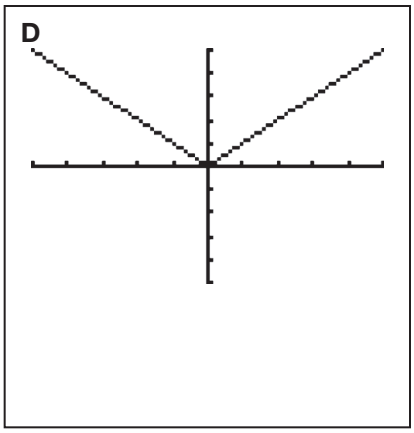
**b.** If possible, identify other graphs that show vertical symmetry.

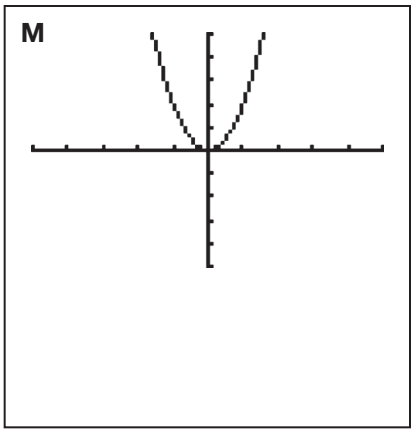
© 2012 Carnegie Learning

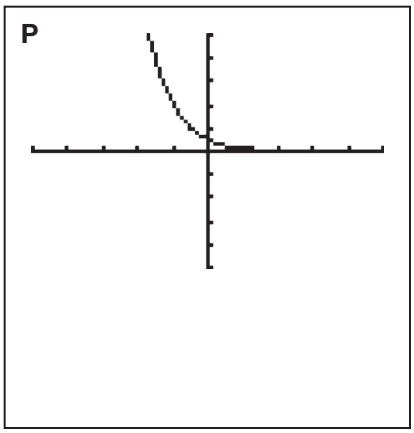
3.

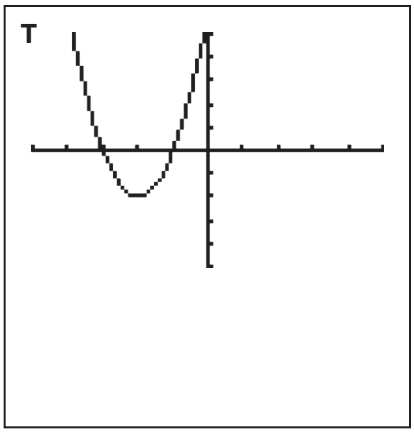
**Duane**

I grouped these graphs together because each graph only goes through two quadrants.



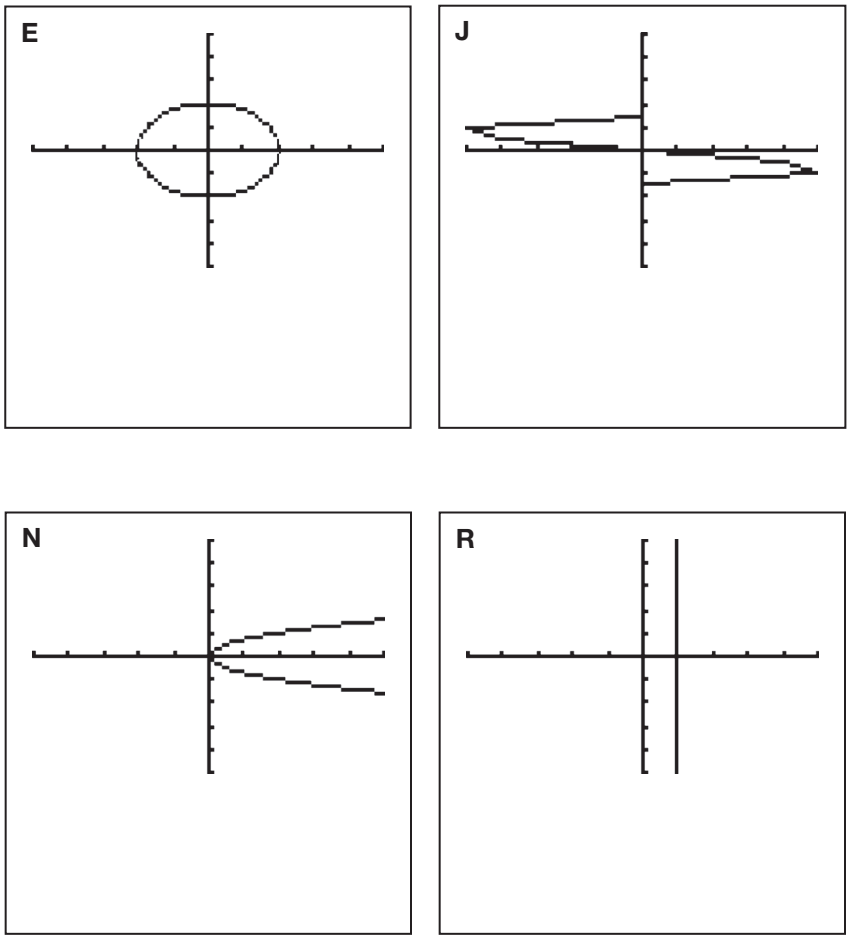




**a.** Explain why Duane's reasoning is not correct.

**b.** If possible, identify other graphs that only go through two quadrants.

© 2012 Carnegie Learning

**4.** Josephine grouped these four graphs together, but did not provide any rationale.




**a.** What do you notice about the graphs?

**b.** What rationale could Josephine have provided?

© 2012 Carnegie Learning

## PROBLEM 3 Function Junction

A **relation** is the mapping between a set of input values called the **domain** and a set of output values called the **range**. A **function** is a relation between a given set of elements, such that for each element in the domain there exists exactly one element in the range.

The **Vertical Line Test** is a visual method used to determine whether a relation represented as a graph is a function. To apply the Vertical Line Test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

A **discrete graph** is a graph of isolated points. A **continuous graph** is a graph of points that are connected by a line or smooth curve on the graph. Continuous graphs have no breaks.

The Vertical Line Test applies for both discrete and continuous graphs.

1. Analyze the four graphs Judy grouped together. Do you think that the graphs she grouped are functions? Explain how you determined your conclusion.

2. Use the Vertical Line Test to sort the graphs in Problem 1 into two groups: functions and non-functions. Record your results by writing the letter of each graph in the appropriate column in the table shown.

| Functions | Non-Functions |
|---|---|
| | |

© 2012 Carnegie Learning

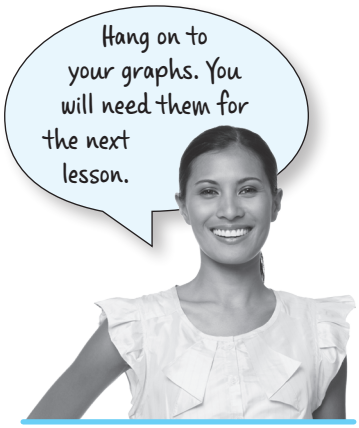
**3.** Each graph in this set of functions has a domain that is either:

- the set of all real numbers, or
- the set of integers.

For each graph, remember that the *x*-axis and the *y*-axis display values from −10 to 10 with an interval of 2 units.

Label each function graph with the appropriate domain.

**4.** Clip all your graphs together and keep them for the next lesson.

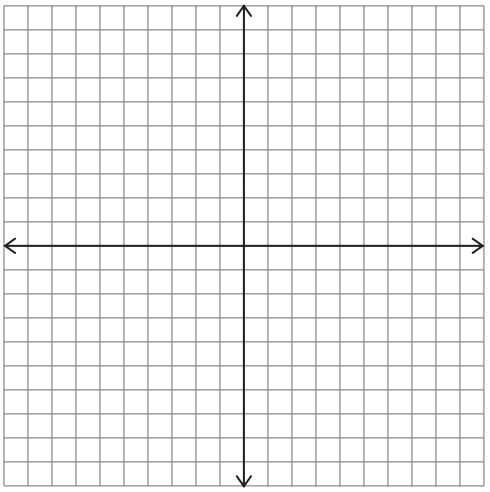



© 2012 Carnegie Learning

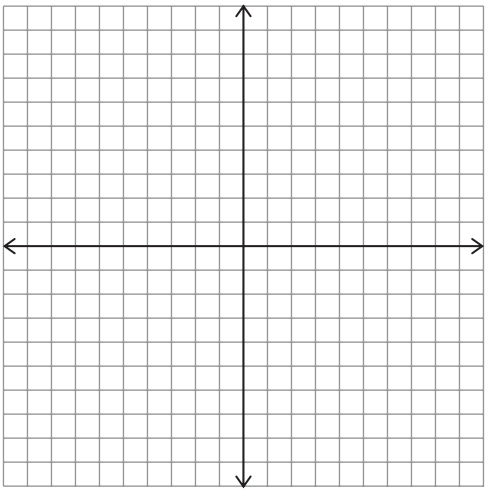
1

## Talk the Talk

**1.** Sketch a graph of a function. Explain how you know that it is a function.



**2.** Sketch a graph that is not a function. Explain how you know that it is not a function.



Be prepared to share your solutions and methods.

© 2012 Carnegie Learning

# There Are Many Ways to Represent Functions

1.3

## Recognizing Algebraic and Graphical Representations of Functions

### LEARNING GOALS

In this lesson, you will:

- Write equations using function notation.
- Recognize multiple representations of functions.
- Determine and recognize characteristics of functions.
- Determine and recognize characteristics of function families.

### KEY TERMS

- function notation
- increasing function
- decreasing function
- constant function
- function family
- linear functions
- exponential functions
- absolute minimum
- absolute maximum
- quadratic functions
- linear absolute value functions
- linear piecewise functions

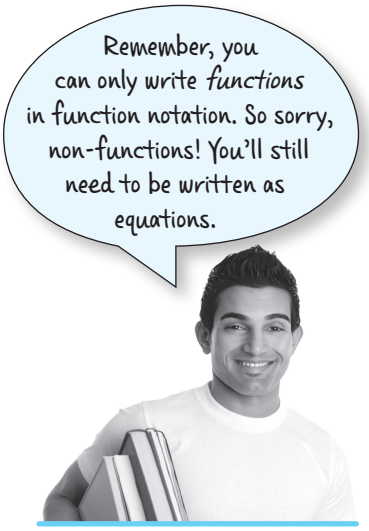
Just about everything you see, hear, or own has a name. It's not just people who have names—streets have names, cars have names, even trees have names. So where do these names come from and why were they chosen? There are many naming conventions we use in our society. The purpose of these naming conventions is to provide useful information about the object being named. For example, just saying "I live on a street" does not provide much information. However, saying "I live on East Main Street" makes it much more clear where you live.

Think about other objects and their names. Why do you think they were named the way they were? What information is provided by their names? Would another name suit the object better?

© 2012 Carnegie Learning

## PROBLEM 1 A New Way to Write Something Familiar

Functions can be represented in a number of ways. An equation representing a function can be written using *function notation*. **Function notation** is a way of representing functions algebraically. This form allows you to more efficiently identify the independent and dependent quantities. The function $f(x)$ is read as "$f$ of $x$" and indicates that $x$ is the independent variable.




Let's look at the relationship between an equation and function notation.

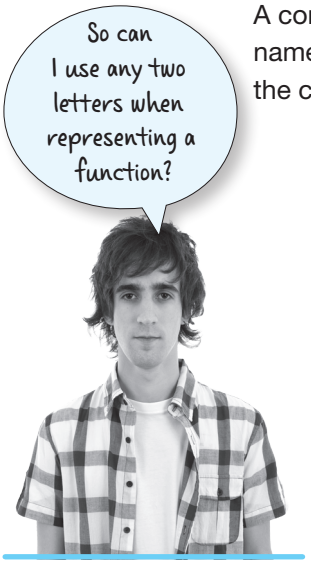
Consider orders for a custom T-shirt shop. U.S. Shirts charges \$8 per shirt plus a one-time charge of \$15 to set a T-shirt design. The equation $y = 8x + 15$ can be written to model this situation. The independent variable $x$ represents the number of shirts ordered, and the dependent variable $y$ represents the total cost of the order, in dollars.

You know this is a function because for each number of shirts ordered (independent value) there is exactly one total cost (dependent value) associated with it.

Because this situation is a function, you can write $y = 8x + 15$ in function notation.

$$f(x) = 8x + 15$$

The cost, defined by $f$, is a function of $x$, the number of shirts ordered.




A common way to name a function is $f(x)$. However, you can choose any variable to name a function. You could write the T-shirt cost function as $C(s) = 8s + 15$, where the cost, defined as $C$, is a function of $s$, the number of shirts ordered.

© 2012 Carnegie Learning

You can input equations written in function notation into your graphing calculator. Your graphing calculator will list different functions as $\mathbf{Y_1}$, $\mathbf{Y_2}$, $\mathbf{Y_3}$, etc.

Let's graph the function $f(x) = 8x + 15$ on a calculator by following the steps shown.

You can use a graphing calculator to graph a function.

**Step 1:** Press **Y=**. Your cursor should be blinking on the line **\Y1=**. Enter the equation. To enter a variable like $x$, press the key with **X, T, Θ, n** once.

**Step 2:** Press **WINDOW** to set the bounds and intervals you want displayed.

**Step 3:** Press **GRAPH** to view the graph.

The way you set the window will vary each time depending on the equation you are graphing.

```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-20
 Ymax=20
 Yscl=2
```

The **Xmin** represents the least point on the $x$-axis that will be seen on the screen. The **Xmax** represents the greatest point that will be seen on the $x$-axis. Lastly, the **Xscl** represents the intervals. Similar names are used for the $y$-axis (**Ymin, Ymax,** and **Yscl**).

A convention to communicate the viewing **WINDOW** on a graphing calculator is shown.

**Xmin:** $-10$, **Xmax:** $10$ $\rightarrow$ $[-10, 10]$

**Ymin:** $-20$, **Ymax:** $20$ $\rightarrow$ $[-20, 20]$

$[-10, 10] \times [-20, 20]$

© 2012 Carnegie Learning

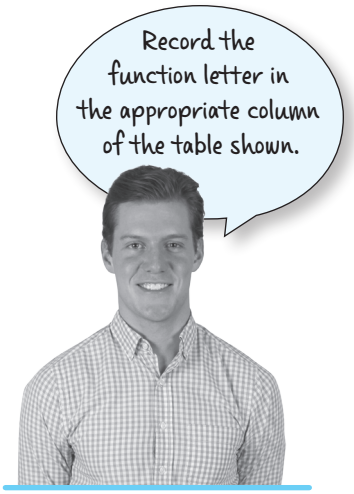
## PROBLEM 2 Up, Down, or Neither?

In the previous lesson, you determined which of the given graphs represented functions. Gather all of the graphs from the previous lesson that you identified as functions.

A function is described as increasing when the dependent variable increases as the independent variable increases. If a function increases across the entire domain, then the function is called an **increasing function**.

A function is described as decreasing when the dependent variable decreases as the independent variable increases. If a function decreases across the entire domain, then the function is called a **decreasing function**.

If the dependent variable of a function does not change or remains constant over the entire domain, then the function is called a **constant function**.



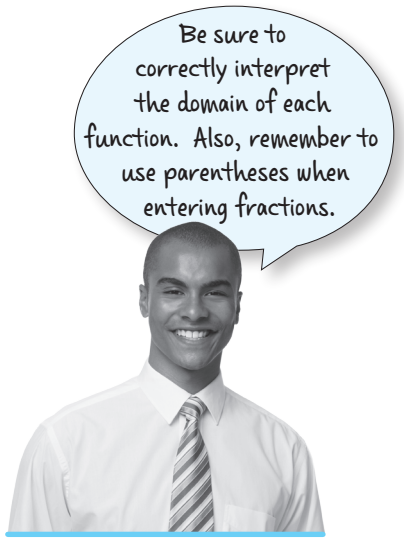

1. Analyze each graph from left to right. Sort all the graphs into one of the four groups:
   - increasing function,
   - decreasing function,
   - constant function,
   - a combination of increasing, decreasing, or constant.

| Increasing Function | Decreasing Function | Constant Function | Combination of Increasing, Decreasing, or Constant |
|---|---|---|---|
| | | | |

© 2012 Carnegie Learning

**2.** Each function shown represents one of the graphs in the increasing function, decreasing function, or constant function categories. Enter each function into a graphing calculator to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.

- $f(x) = x$
- $f(x) = \left(\frac{1}{2}\right)^x - 5$
- $f(x) = 2^x$, where $x$ is an integer
- $f(x) = -\frac{2}{3}x + 5$
- $f(x) = -x + 3$, where $x$ is an integer
- $f(x) = \left(\frac{1}{2}\right)^x$
- $f(x) = 2$, where $x$ is an integer




**3.** Consider the seven graphs and functions that are increasing functions, decreasing functions, or constant functions.

**a.** Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

| Group 1 | Group 2 |
|---|---|
| | |

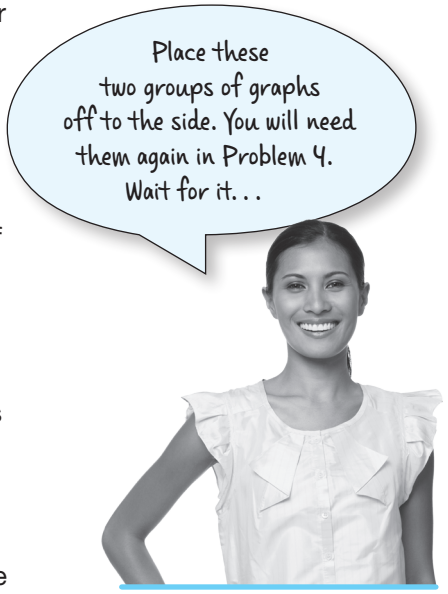
**b.** What is the same about all the functions in each group?

© 2012 Carnegie Learning

Congratulations! You have just sorted the graphs into their own *function families.* A **function family** is a group of functions that share certain characteristics.

The family of **linear functions** includes functions of the form $f(x) = mx + b$, where $m$ and $b$ are real numbers.

The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x$, where $a$ and $b$ are real numbers, and $b$ is greater than 0 but is not equal to 1.




4. Go back to your table in Question 3 and identify which group represents linear and constant functions and which group represents exponential functions.

5. If $f(x) = mx + b$, represents a linear function, describe the $m$ and $b$ values that produce a constant function.

## PROBLEM 3 Least, Greatest, or Neither?

A function has an **absolute minimum** if there is a point that has a $y$-coordinate that is less than the $y$-coordinates of every other point on the graph. A function has an **absolute maximum** if there is a point that has a $y$-coordinate that is greater than the $y$-coordinates of every other point on the graph.

1. Sort the graphs from the Combination category in Problem 2 into three groups:
   - those that have an absolute minimum value,
   - those that have an absolute maximum value, and
   - those that have no absolute minimum or maximum value.

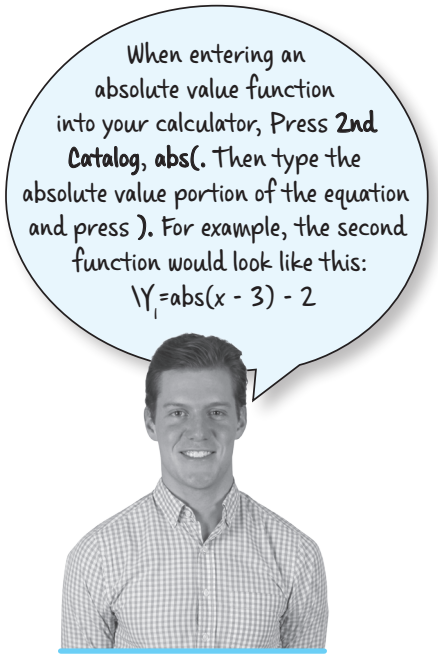
   Then record the function letter in the appropriate column of the table shown.

| Absolute Minimum | Absolute Maximum | No Absolute Minimum or Absolute Maximum |
|---|---|---|
| | | |

© 2012 Carnegie Learning

**2.** Each function shown represents one of the graphs with an absolute maximum or an absolute minimum value. Enter each function into your graphing calculator to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.

- $f(x) = x^2 + 8x + 12$
- $f(x) = |x - 3| - 2$
- $f(x) = x^2$
- $f(x) = |x|$
- $f(x) = -|x|$
- $f(x) = -3x^2 + 4$, where $x$ is integer
- $f(x) = -\frac{1}{2}x^2 + 2x$
- $f(x) = -2|x + 2| + 4$




**3.** Consider the graphs of functions that have an absolute minimum or an absolute maximum. (Do not consider Graphs A and C yet.)

**a.** Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

| Group 1 | Group 2 |
|---|---|
| | |

**b.** What is the same about all the functions in each group?

© 2012 Carnegie Learning

Congratulations! You have just sorted functions into two more function families.

The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where $a$, $b$, and $c$ are real numbers, and $a$ is not equal to 0.

The family of **linear absolute value functions** includes functions of the form $f(x) = a|x + b| + c$, where $a$, $b$, and $c$ are real numbers, and $a$ is not equal to 0.

**4.** Go back to your table in Question 3 and identify which group represents quadratic functions and which group represents linear absolute value functions.

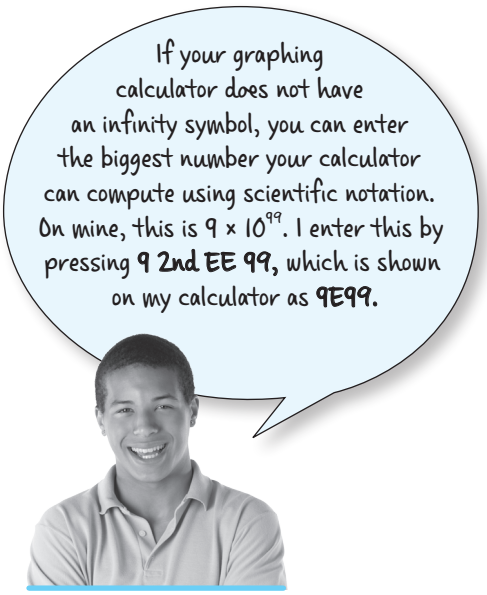
## PROBLEM 4 Piecing Things Together

Analyze each of the functions shown. These functions represent the last three graphs of functions from the no absolute minimum and no absolute maximum category.

- $f(x) = \begin{cases} -2x + 10, & -\infty \le x < 3 \\ 4, & 3 \le x < 7 \\ -2x + 18, & 7 \le x \le +\infty \end{cases}$

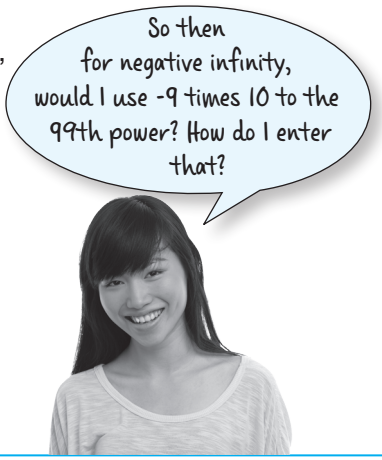
- $f(x) = \begin{cases} -2, & -\infty < x < 0 \\ \frac{1}{2}x - 2, & 0 \le x < \infty \end{cases}$

- $f(x) = \begin{cases} \frac{1}{2}x + 4, & -\infty \le x < 2 \\ -3x + 11, & 2 \le x < 3 \\ \frac{1}{2}x + \frac{1}{2} & 3 \le x \le \infty \end{cases}$




These functions are part of the family of *linear piecewise functions*. **Linear piecewise functions** include functions that have equation changes for different parts, or pieces, of the domain.

Because these graphs each contain compound inequalities, there are additional steps required to use a graphing calculator to graph each function.




© 2012 Carnegie Learning

Let's graph the piecewise function:

$$\begin{cases} -2x + 10, & -\infty \le x < 3 \\ f(x) = 4, & 3 \le x < 7 \\ f(x) = -2x + 18, & 7 \le x \le +\infty \end{cases}$$

You can use a graphing calculator to graph piecewise functions.

**Step 1:** Press **Y=**. Enter the first section of the function within parentheses. Then press the division button.

The first section is $(-2x + 10)$.

**Step 2:** Press the ( key twice and enter the first part of the compound inequality within parentheses.

The first part of the inequality is $(-\infty \le x)$.

Use the **2ND TEST** to locate the inequality symbols. The **TEST** button is on the same key as the **MATH** button.

**Step 3:** Enter the second part of the compound inequality within parentheses and then type two closing parentheses.

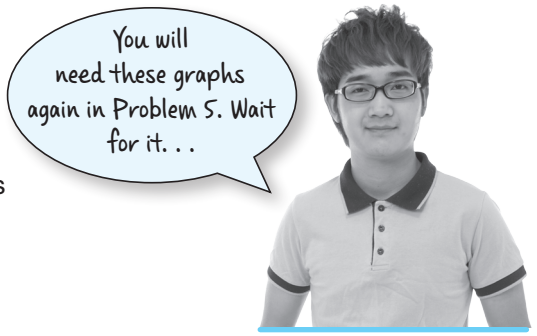
Press **GRAPH** here to see the first section of the piecewise function.

The second part of the compound inequality is $(x < 3)$. The entire line should look like: $\backslash Y_1 = (-2x + 10)/((-\infty \le x)(x < 3))$

**Step 4:** Enter the remaining sections of the piecewise functions as $Y_2$ and $Y_3$.

By completing the first piecewise function, you can now choose the graph that matches your graphing calculator screen.

1. Enter the remaining functions into your graphing calculator to determine the shapes of their graphs.
2. Match each function to its corresponding graph by writing the function directly on the graph that it represents.




© 2012 Carnegie Learning

1

Congratulations! You have just sorted the remaining functions into one more function family.

The family of linear piecewise functions includes functions that have equation changes for different parts, or pieces, of the domain.

© 2012 Carnegie Learning

## PROBLEM 5 We Are Family!

You have now sorted each of the graphs and equations representing functions into one of five function families: linear, exponential, quadratic, linear absolute value, and linear piecewise.

1. Glue your sorted graphs and functions to the appropriate function family Graphic Organizer on the pages that follow. Write a description of the graphical behavior for each function family.

You've done a lot of work up to this point! You've been introduced to linear, exponential, quadratic, linear absolute value, and linear piecewise functions. Don't worry—you don't need to know everything there is to know about all of the function families right now. As you progress through this course, you will learn more about each function family.

Be prepared to share your solutions and methods.

© 2012 Carnegie Learning

© 2012 Carnegie Learning

## Definition

The family of **linear functions** includes functions of the form $f(x) = mx + b$, where $m$ and $b$ are real numbers.

## Graphical Behavior

**Increasing / Decreasing / Constant:**

**Maximum / Minimum:**

**Curve / Line:**

# Linear Functions

## Examples

© 2012 Carnegie Learning

1

## Definition

The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x$, where $a$ and $b$ are real numbers, and $b$ is greater than 0 but not equal to 1.

## Graphical Behavior

**Increasing / Decreasing / Constant:**

**Maximum / Minimum:**

**Curve / Line:**

# Exponential Functions

## Examples

© 2012 Carnegie Learning

## Definition

The family of **quadratic functions** includes functions of the form, $f(x) = ax^2 + bx + c$ where $a$, $b$, and $c$ are real numbers, and $a$ is not equal to 0.

## Graphical Behavior

**Increasing / Decreasing / Constant:**

**Maximum / Minimum:**

**Curve / Line:**

# Quadratic Functions

## Examples

© 2012 Carnegie Learning

1

## Linear Absolute Value Functions

### Definition

The family of **linear absolute value functions** includes functions of the form $f(x) = a|x + b| + c$, where $a$, $b$, and $c$ are real numbers, and $a$ is not equal to 0.

### Graphical Behavior

**Increasing / Decreasing / Constant:**

**Maximum / Minimum:**

**Curve / Line:**

### Examples

© 2012 Carnegie Learning

## Definition

The family of **linear piecewise functions** includes functions that have equation changes for different parts, or pieces, of the domain.

## Graphical Behavior

**Increasing / Decreasing / Constant:**

**Maximum / Minimum:**

**Curve / Line:**

# Linear Piecewise Functions

## Examples

© 2012 Carnegie Learning

© 2012 Carnegie Learning

# Function Families for 200, Alex…

## Recognizing Functions by Characteristics

1.4

### LEARNING GOALS

In this lesson, you will:

- Recognize similar characteristics among function families.
- Recognize different characteristics among function families.
- Determine function types given certain characteristics.

Since the debut of television in the early 1950s, Americans have had a love/hate relationship with the game show. One of the original game shows that aired was *Name that Tune*. The game was played when two contestants were given a clue about a song. Then, one opponent would "bid" that the song could be named in a certain number of notes played. The other opponent could either beat the number of notes "bid" from the opponent, or they could tell their opponent to "name that tune!"

Do you like game shows? If so, what are your favorite game shows?

© 2012 Carnegie Learning

1

## PROBLEM 1 Name That Function!

**1.** Use the characteristic(s) provided to choose the appropriate function family or families from the word box shown.

| | |
|---|---|
| linear function family | exponential function family |
| quadratic function family | linear absolute value function family |

**a.** The graph of this function family:
- is a smooth curve.

**b.** The graph of this function family:
- is made up of one or more straight lines.

**c.** The graph of this function family:
- increases or decreases over the entire domain.

**d.** The graph of this function family:
- has a maximum or a minimum.

**2.** A second characteristic has been added to the graphical description of each function. Name the possible function family or families given the graphical characteristics.

**a.** The graph of this function family:
- has an absolute minimum or absolute maximum, and
- is a smooth curve.

**b.** The graph of this function family:
- either increases or decreases over the entire domain, and
- is made up of a straight line.

© 2012 Carnegie Learning

**c.** The graph of this function family:

- is a smooth curve, and
- either increases or decreases over the entire domain.

**d.** The graph of this function family:

- has either an absolute minimum or an absolute maximum
- has symmetry, and
- is made up of 2 straight lines

Each function family has certain graphical behaviors with some behaviors common among different function families. Notice, the more specific characteristics that are given, the more specifically you can Name that Function!

© 2012 Carnegie Learning

## PROBLEM 2 Graph That Function!

**1.** Use the given characteristics to create an equation and sketch a graph. Use the equations given in the box as a guide. Then share your graph with your partner. Discuss similarities and differences between your graphs.

When creating your equation, use $a$, $b$, and $c$ values that are any real numbers between $-3$ and 3. Do not use any functions that were used previously in this chapter.

**Linear function**

$f(x) = mx + b$

**Exponential function**

$f(x) = a \cdot b^x$

**Quadratic function**

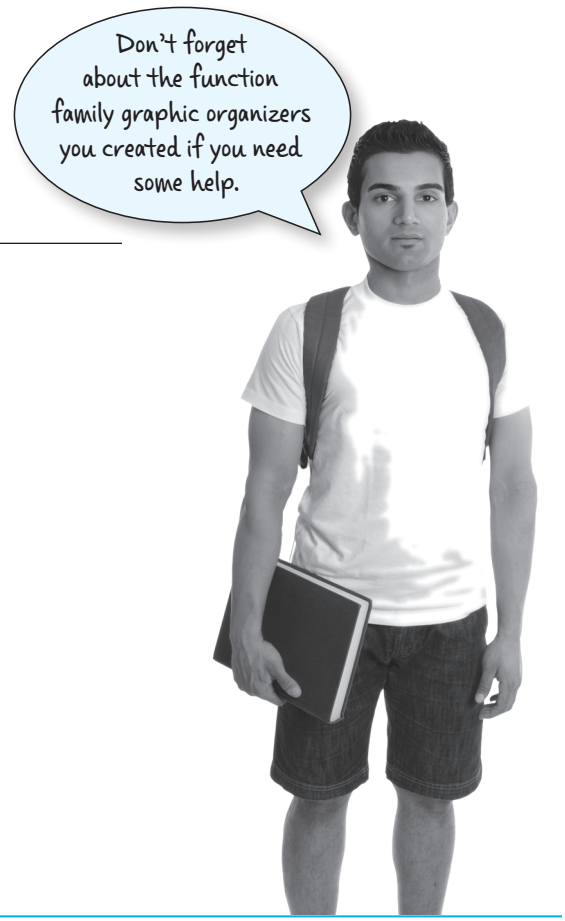
$f(x) = ax^2 + bx + c$

**Linear Absolute Value Function**

$f(x) = a|x + b| + c$

**a.** Create an equation and sketch a graph that:

- is a function,
- is exponential,
- is continuous, and
- is decreasing.



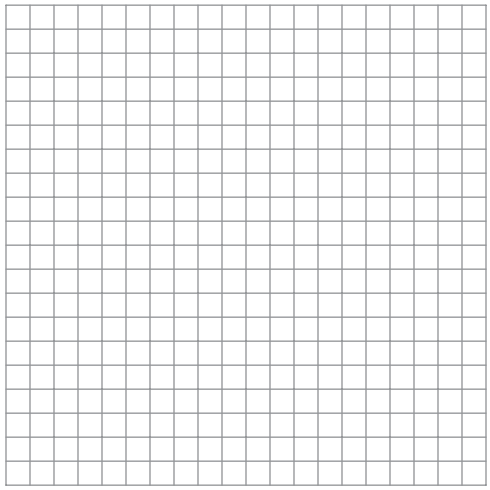

Equation: _____

© 2012 Carnegie Learning

**b.** Create an equation and sketch a graph that:

- has a minimum,
- is discrete, and
- is a linear absolute value function.

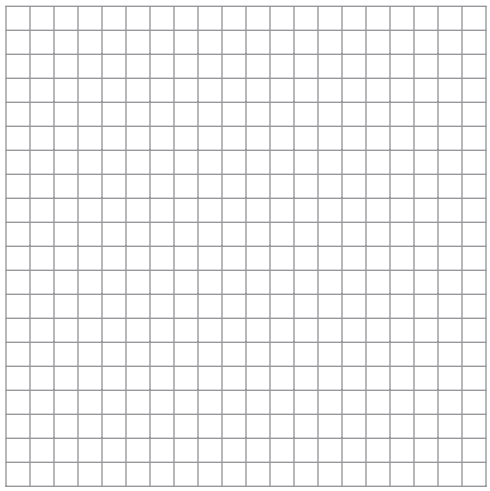
Equation: _____



**c.** Create an equation and sketch a graph that:

- is linear,
- is discrete,
- is increasing, and
- is a function.

Equation: _____

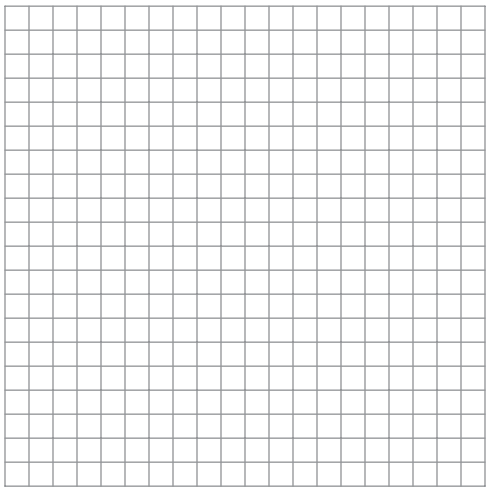


© 2012 Carnegie Learning

**d.** Create an equation and sketch a graph that:

- is continuous,
- has a maximum,
- is a function, and
- is quadratic.

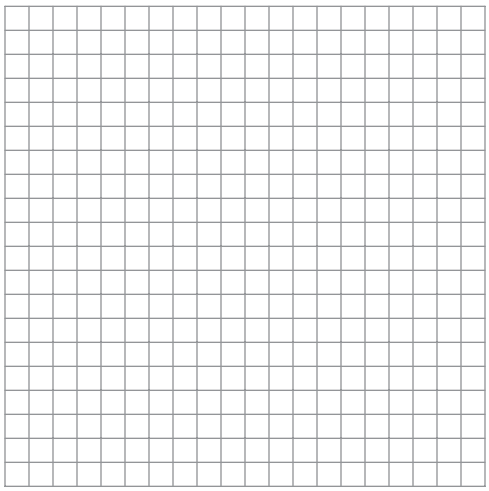
Equation: _____



**e.** Create an equation and sketch a graph that:

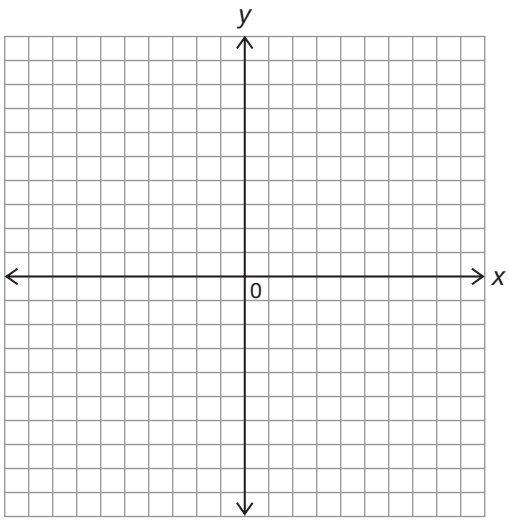
- is not a function,
- is continuous, and
- is a straight line.

Equation: _____



© 2012 Carnegie Learning

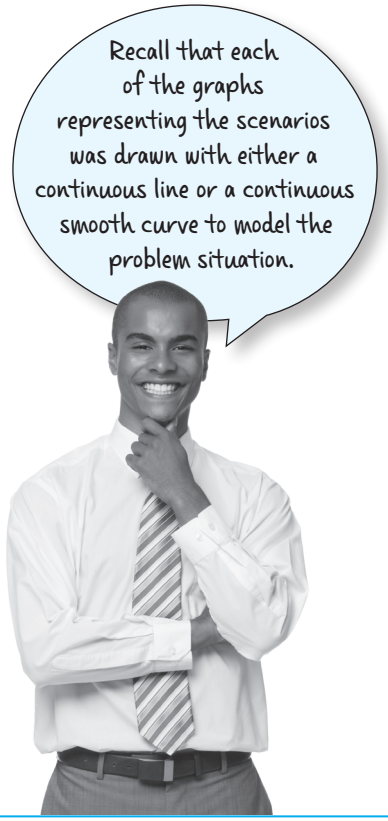
2. Create your own function. Describe certain characteristics of the function and see if your partner can sketch it. Then try to sketch your partner's function based on characteristics provided.




## Talk the Talk

Throughout this chapter, you were introduced to five function families: linear, exponential, quadratic, linear absolute value, and linear piecewise. Let's revisit the first lesson in this chapter: *A Picture Is Worth a Thousand Words*. Each of the scenarios in this lesson represents one of these function families.

1. Describe how each scenario represents a function.




2. Complete the table to describe each scenario.
   a. Identify the appropriate function family.
   b. Based on the problem situation, identify whether the graph of the function should be discrete or continuous.
   c. Create a sketch of the mathematical model.
   d. Identify the graphical behavior.

© 2012 Carnegie Learning

| Scenario | Function Family | Domain of the Real-World Situation: Discrete or Continuous | Sketch of the Mathematical Model | Graphical Behavior | |
|---|---|---|---|---|---|
| | | | | Absolute Minimum or Absolute Maximum | Increasing, Decreasing, Constant, or Combination |
| Something's Fishy | | | | | |
| Smart Phone, but Is It a Smart Deal? | | | | | |
| Can't Wait to Hit the Slopes! | | | | | |
| It's Magic | | | | | |

© 2012 Carnegie Learning

© 2012 Carnegie Learning

Be prepared to share your solutions and methods.

| Scenario | Function Family | Domain of the Real-World Situation: Discrete or Continuous | Sketch of the Mathematical Model | Graphical Behavior | |
|---|---|---|---|---|---|
| | | | | Absolute Minimum or Absolute Maximum | Increasing, Decreasing, Constant, or Combination |
| Baton Twirling | | | | | |
| Music Club | | | | | |
| A Trip to School | | | | | |
| Jelly Bean Challenge | | | | | |

1

© 2012 Carnegie Learning

# Chapter 1 Summary

## KEY TERMS

- dependent quantity (1.1)
- independent quantity (1.1)
- relation (1.2)
- domain (1.2)
- range (1.2)
- function (1.2)
- Vertical Line Test (1.2)
- discrete graph (1.2)
- continuous graph (1.2)
- function notation (1.3)
- increasing function (1.3)
- decreasing function (1.3)
- constant function (1.3)
- function family (1.3)
- linear functions (1.3)
- exponential functions (1.3)
- absolute minimum (1.3)
- absolute maximum (1.3)
- quadratic functions (1.3)
- linear absolute value functions (1.3)
- linear piecewise functions (1.3)

## 1.1 Identifying the Dependent and Independent Quantities for a Problem Situation

Many problem situations include two quantities that change. When one quantity depends on another, it is said to be the dependent quantity. The quantity that the dependent quantity depends upon is called the independent quantity.

### Example

Caroline makes $8.50 an hour babysitting for her neighbors' children after school and on the weekends.

The dependent quantity is the total amount of money Caroline earns based on the independent quantity. The independent quantity is the total number of hours she babysits.

© 2012 Carnegie Learning

1

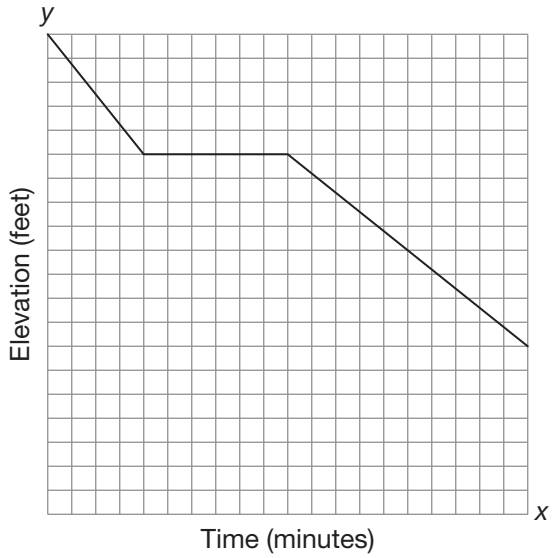
## 1.1 Labeling and Matching a Graph to an Appropriate Problem Situation

Graphs relay information about data in a visual way. Connecting points on a coordinate plane with a line or smooth curve is a way to model or represent relationships. The independent quantity is graphed on the horizontal or *x*-axis, while the dependent quantity is graphed on the vertical, or *y*-axis. Graphs can be straight lines or curves, and can increase or decrease from left to right. When matching with a problem situation, consider the situation and the quantities to interpret the meaning of the data values.

### Example

Pedro is hiking in a canyon. At the start of his hike, he was at 3500 feet. During the first 20 minutes of the hike, he descended 500 feet at a constant rate. Then he rested for half an hour before continuing the hike at the same rate.

Time is the independent quantity and elevation is the dependent quantity.



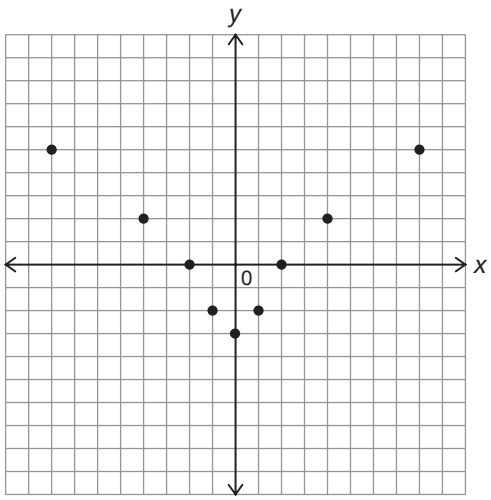

## 1.2 Analyzing and Comparing Types of Graphs

Looking for patterns can help when sorting and comparing graphs. A discrete graph is a graph of isolated points. A continuous graph is a graph of points with no breaks in it. The points are connected by a straight line or smooth curve. Some graphs show vertical symmetry (if a vertical line were drawn through the middle of the graph the image is the same on both sides). Other possible patterns to look for include: only goes through two quadrants, always increasing from left to right, always decreasing from left to right, straight lines, smooth curves, the graph goes through the origin, the graph forms a U shape, the graph forms a V shape.

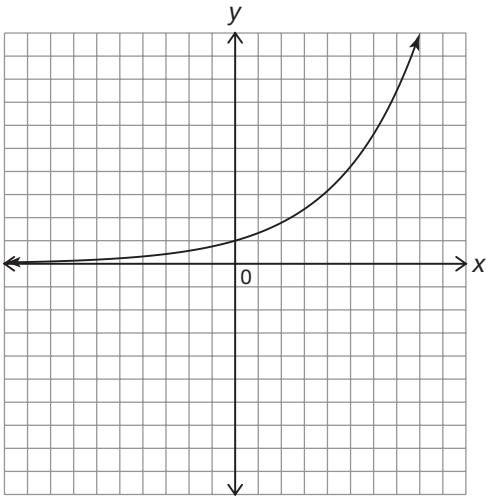
© 2012 Carnegie Learning

### Examples

A discrete graph with vertical symmetry:




A continuous graph with a smooth curve increasing from left to right:

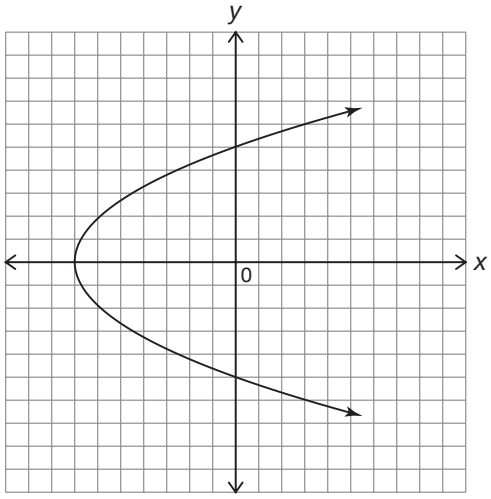



## 1.2 Using the Vertical Line Test When Determining Whether a Relation Is a Function

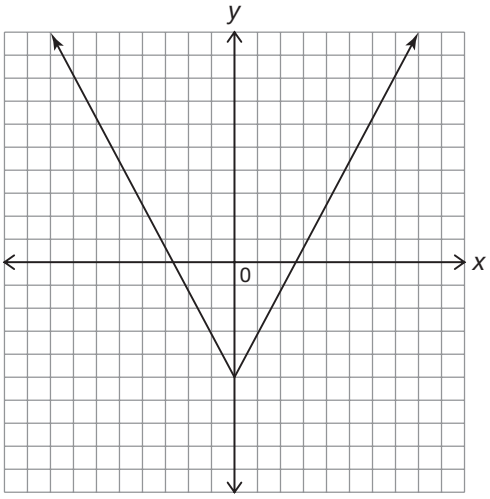
A relation is the mapping between a set of input values called the domain and a set of output values called the range. A function is a relation between a given set of elements for which each element in the domain has exactly one element in the range. The Vertical Line Test is a visual method used to determine whether a relation represented as a graph is a function. To apply the Vertical Line Test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

### Examples

A line drawn vertically through the graph touches more than one point. The graph does not represent a function.




A line drawn vertically through the graph only touches one point. The graph represents a function.




© 2012 Carnegie Learning

## 1.3 Writing Equations Using Function Notation

Functions can be represented in a number of ways. An equation representing a function can be written using function notation. Function notation is a way of representing functions algebraically. This form allows you to more efficiently identify the independent and dependent quantities. The function $f(x)$ is read as "$f$ of $x$" and indicates that $x$ is the independent variable. Remember, you know an equation is a function because for each independent value there is exactly one dependent value associated with it.

### Example

Write this equation using function notation:
$y = 2x + 5$

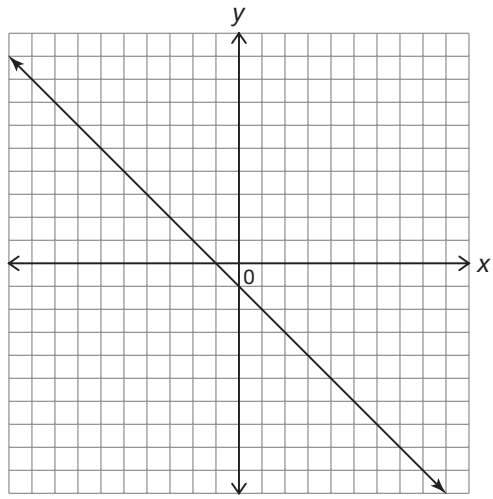
The dependent variable ($y$), defined by $f$, is a function of $x$, the independent variable.
$f(x) = 2x + 5$

## 1.3 Determining Whether a Graph Represents a Function That Is Increasing, Decreasing, or Constant

A function is described as increasing when both the independent and dependent variables are increasing. If a function increases across the entire domain, then the function is called an increasing function. A function is described as decreasing when the dependent variable decreases as the independent variable increases. If a function decreases across the entire domain, then the function is called a decreasing function. If the dependent variable of a function does not change or remains constant over the entire domain, then the function is called a constant function.

### Example

The function shown in the graph is a decreasing function because the dependent variable ($y$) decreases as the independent variable ($x$) increases.



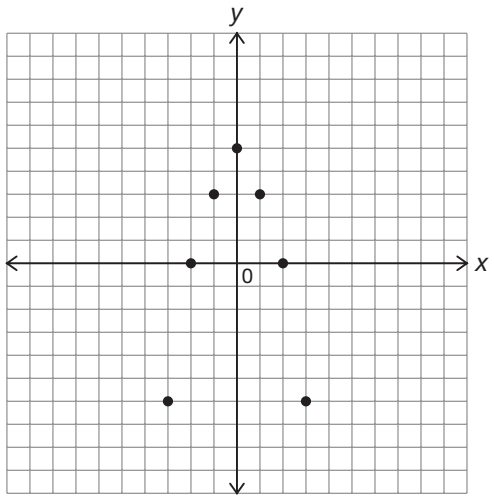

© 2012 Carnegie Learning

## 1.3 Determining Whether a Graph Represents a Function with an Absolute Maximum or Absolute Minimum

A function has an absolute minimum if there is a point that has a *y*-coordinate that is less than the *y*-coordinates of every other point on the graph. A function has an absolute maximum if there is a point that has a *y*-coordinate that is greater than the *y*-coordinates of every other point on the graph.

### Example

The function shown in the graph has an absolute maximum because the *y*-coordinate of the point (0, 5) is greater than the *y*-coordinates of every other point on the graph.




## 1.3 Distinguishing Between Function Families

A function family is a group of functions that share certain characteristics.

The family of linear functions includes functions of the form $f(x) = ax + b$, where $a$ and $b$ are real numbers.

The family of exponential functions includes functions of the form $f(x) = a \cdot b^x$, where $a$ and $b$ are real numbers, and $b$ is greater than 0, but not equal to 1.

The family of quadratic functions includes functions of the form $f(x) = ax^2 + bx + c$, where $a$, $b$, and $c$ are real numbers, and $a$ is not equal to 0.

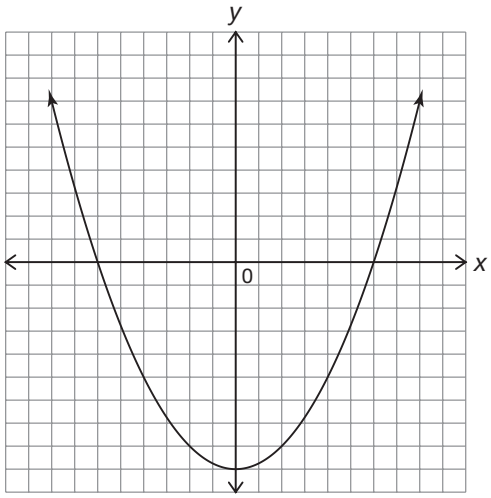
The family of linear absolute value functions includes functions of the form $f(x) = a|x + b| + c$, where $a$, $b$, and $c$ are real numbers, and $a$ is not equal to 0.

The family of linear piecewise functions includes functions that has an equation that changes for different parts, or pieces, of the domain.

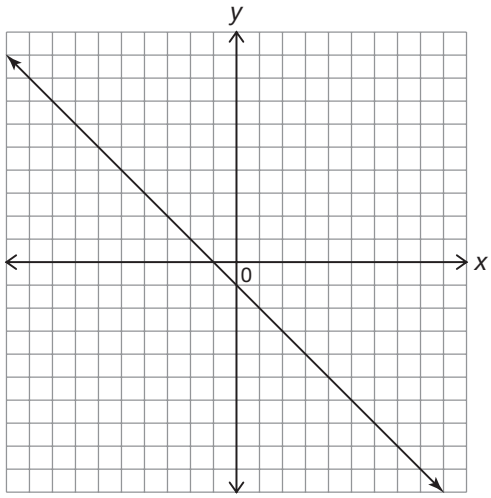
© 2012 Carnegie Learning

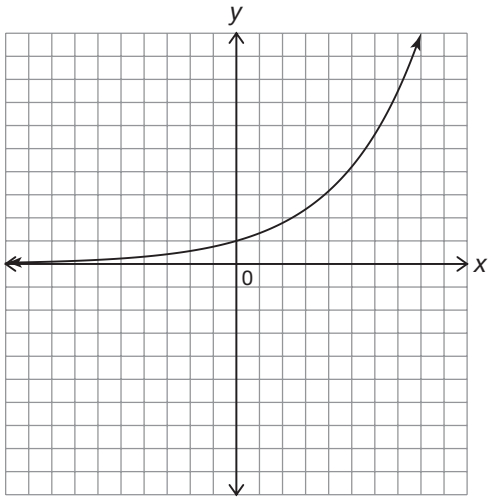
## Examples

The function is quadradic.

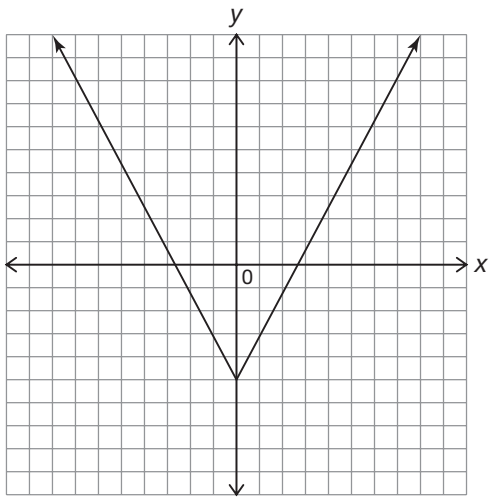



The function is linear.

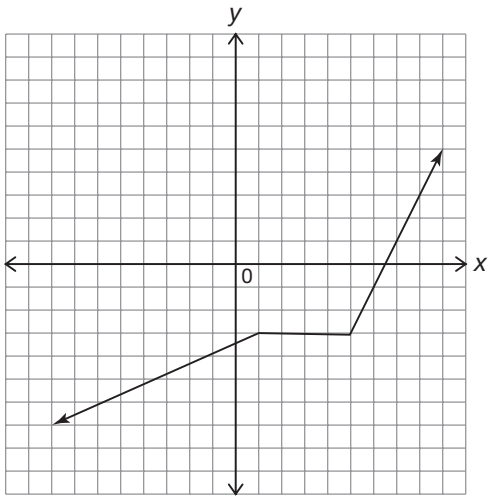



The function is exponential.




The function is linear absolute value.




The function is linear piecewise.

© 2012 Carnegie Learning

## 1.4 Identifying a Function Given Its Characteristics

Certain characteristics of a graph such as whether it increases or decreases over its domain, has an absolute minimum or maximum, is a smooth curve or not, or other characteristics, can help when determining if a function is linear, exponential, quadratic, or linear absolute value.

### Example

The graph of a function $f(x)$ is a smooth curve and has an absolute minimum. Thus, the function is quadradtic.

## 1.4 Graphing a Function Given Its Characteristics

Use the given characteristics to create an equation and sketch a graph.

Linear function $f(x) = mx + b$

Exponential function $f(x) = a \cdot b^x$

Quadratic function $f(x) = ax^2 + bx + c$

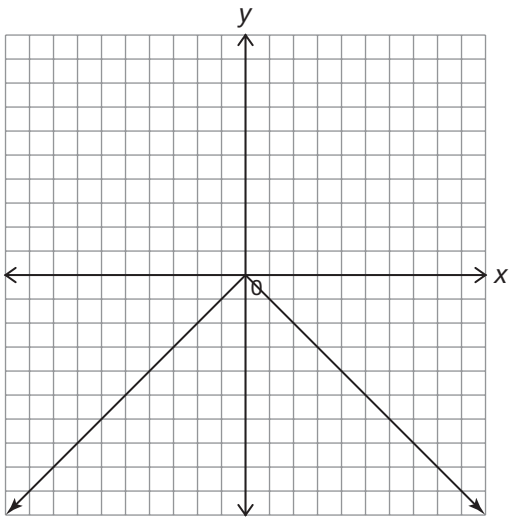
Linear Absolute Value Function $f(x) = a|x + b| + c$

### Example

Create an equation and sketch a graph that has:

- an absolute maximum
- and is a linear absolute value function.

$f(x) = -|x|$



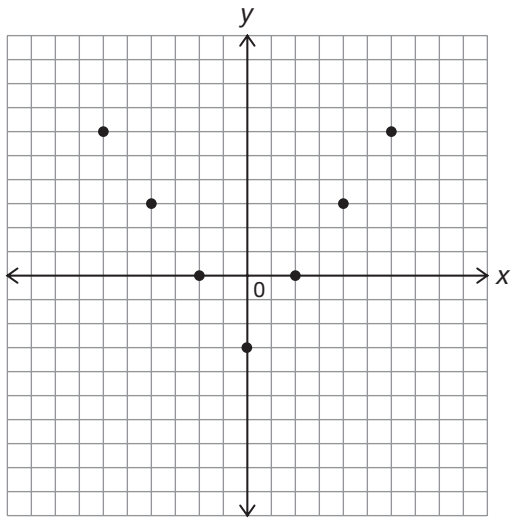

© 2012 Carnegie Learning

## 1.4 Identifying a Function Given Its Graph

Certain characteristics of a graph such as whether it increases or decreases over its domain, has an absolute minimum or maximum, is a smooth curve or not, or other characteristics, can help when determining if a graph represents a linear, exponential, quadratic, or linear absolute value function.

### Example

The graph shown is a linear absolute value function. It is discrete. The graph decreases and then increases. It has an absolute minimum.




© 2012 Carnegie Learning